

Ghosts and Stability of Asymptotically Safe Gravity in the Minkowski Background

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We investigate the problems of ghosts and stability in the framework of asymptotically safe theory of gravity in the Minkowski background. Within one-loop corrections, we obtain explicitly the constraints on the coupling constants. Applying them to the ones recently-obtained at the fixed point, we find that the corresponding theory is both ghost-free and stable. Our results can be easily generalized to high-loop corrections.

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I. INTRODUCTION

In 1974, 't Hooft and Veltman [1] showed that General Relativity is notoriously non-renormalizable at the quantum level. Further extensive investigations from other researchers [2–7] confirmed this fact. In view of it, Weinberg proposed an effective quantum theory of gravity that turns out to be ultraviolet complete and nonperturbatively renormalizable, because it is endowed with asymptotically safe property [8]. It is found that the renormalization group flows have a stable fixed point in the ultraviolet limit and a finite dimensional critical surface of trajectories appears in the vicinity of this fixed point. This is very interesting and the theory has attracted extensive studies [9–17]. Very recent investigations [18–23] reveal that the ultraviolet critical surface may be only three dimensional even for truncations of the exact renormalization group equations which may have more than three coupling constants. There are two classes of stable fixed points. If the coupling constants vanish at the fixed point, then the fixed point is called Gaussian. On the other hand, if the coupling constants do not vanish at the fixed point, then the point is called non-Gaussian. At the non-Gaussian fixed point, all the coupling constants are non-zero. So the non-Gaussian fixed point is very much physically important. It is found that with the approaching of trajectories to the fixed point, the energy-dependence property of the coupling constants becomes more and more negligible. Finally, the trajectories of motion are irrelevant to the cutoff energy scale.

The aim of this paper is to investigate the ghosts and stability for the asymptotically safe gravity theory with the specific non-Gaussian point. We derive the constraints at the non-Gaussian point that the theory is required to be ghost-free and stable. The paper is organized as follows. Section II presents a brief review of asymptotical safety. In section III, we investigate the

problems of ghosts and stability for the asymptotically safe gravity in the Minkowski background. In Section VI we present our main conclusions. We shall use the system of units with $G = c = \hbar = k_B = 1$ and the metric signature $(-, +, +, +)$ throughout the paper.

II. BRIEF REVIEW OF ASYMPTOTICALLY SAFE GRAVITY

Taking account of multi-loop quantum corrections, the effective action for asymptotically safe theory of gravity takes the form

$$S = \int d^4x \sqrt{-g} [\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R + g_2(\Lambda) R_{\mu\nu} R^{\mu\nu} + g_3(\Lambda) R^2 + g_4(\Lambda) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \Lambda^{-2} g_5 R \square R + \Lambda^{-2} g_6 (\nabla_\mu R^\beta_\alpha) (\nabla^\mu R^\alpha_\beta) + \dots] . \quad (1)$$

Here g denotes the determinant of the metric tensor $g_{\mu\nu}$, R the Ricci scalar, $R_{\mu\nu}$ the Ricci tensor, $R_{\mu\nu\rho\sigma}$ the Riemann tensor, $\square \equiv \nabla_\mu \nabla^\mu$ the four dimensional d'Alembertian operator and Λ the momentum cutoff. The coefficients g_i ($i = 0, 1, 2, 3$) are all dimensionless coupling constants which are assumed to be momentum dependant. In particular, for small momentum we have

$$\Lambda^4 g_0(\Lambda) = \frac{\lambda(\Lambda)}{8\pi}, \quad \Lambda^2 g_1(\Lambda) = \frac{1}{16\pi G(\Lambda)}, \quad (2)$$

where $\lambda(\Lambda)$ and $G(\Lambda)$ correspond to the Einstein cosmological and Newtonian constants, respectively. These coupling constants satisfy the following renormalization group equations:

$$\Lambda \frac{d}{d\Lambda} g_n(\Lambda) = \beta_n[g_i(\Lambda)] . \quad (3)$$

The condition for a fixed point at $g_n = g_{n*}$ is that $\beta_n(g_*) = 0$ for arbitrary n . On the other hand, the conditions for the coupling constants to be attracted to a fixed point at $\Lambda = \infty$ can be achieved by investigating the behavior of $g_n(\Lambda)$ when it is very close to g_{n*} . Assuming that β_n is an analytic function in the vicinity of

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g_{n*} , we can then expand them in Taylor series:

$$\beta_n(g) = \sum_m \left(\frac{\partial \beta_n}{\partial g_m} \right)_* (g_m - g_{*m}) . \quad (4)$$

So, the solution of renormalization group equation is given by

$$g_n(\Lambda) = g_{*n} + \sum_N u_n^N \Lambda^{\zeta_N} , \quad (5)$$

where u^N and ζ_N are the eigenvectors and eigenvalues of the matrix B_{nm} :

$$\sum_m B_{nm} u_m^N = \zeta_N u_n^N . \quad (6)$$

Physically, it is required that the real part of the eigenvalues be negative such that the coupling constants can approach the fixed point. In Ref. [20], by using the exact renormalization group equations, a non-Gaussian fixed point is found to be

$$\begin{aligned} g_{0*} &= 0.0042 , \quad g_{1*} = -0.0101 , \\ g_{2*} &= -0.010 , \quad g_{3*} = 0.0109 . \end{aligned} \quad (7)$$

The goal of this paper is to show, with this new results, there is no ghosts and tachyon for the asymptotically safe gravity in the background of the Minkowski spacetime. We derive the constraints on the coupling constants that the theory is ghost-free and stable.

III. GHOSTS AND STABILITY

For our purpose, let's first consider one-loop corrections, and later we shall generalize our results to high-loop cases. The effective action for asymptotically safe gravity with one-loop corrections is given by,

$$S = \int d^4x \sqrt{-g} (\Lambda^2 g_1 R + g_2 R_{\mu\nu} R^{\mu\nu} + g_3 R^2) . \quad (8)$$

Since we are going to work in the Minkowski background, we are forced to put $g_0 = 0$. On the other hand, due to the Gauss-Bonnet theorem, we can express $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$ as the linear combination of R^2 and $R_{\mu\nu} R^{\mu\nu}$. So for the one-loop action, it is sufficient to consider only the g_2 and g_3 terms. In the Minkowski background, the metric takes the form,

$$ds^2 = -(1 + 2\varepsilon\Phi) dt^2 + (1 - 2\varepsilon\Psi) dx^i dx_i , \quad (9)$$

where Φ and Ψ are the gravitational potentials. It is convenient to introduce the dimensionless constant ε which actually represents the order of perturbations. In order to find the condition to be ghost-free, we should work out the perturbations to the second order in terms of Φ and

Ψ . Then, we have the following expressions:

$$\begin{aligned} \sqrt{-g} &= 1 + \varepsilon(\Phi - 3\Psi) + \varepsilon^2 \left(\frac{3}{2}\Psi^2 - 3\Phi\Psi - \frac{1}{2}\Phi^2 \right) , \\ R &= -\varepsilon \left(2\nabla^2\Phi + 6\ddot{\Psi} - 4\nabla^2\Psi \right) + \varepsilon^2 \left[4\nabla^2\Phi(\Phi - \Psi) \right. \\ &\quad \left. + 16\Psi\nabla^2\Psi + 2(\nabla\Phi \cdot \nabla\Phi + \nabla\Phi \cdot \nabla\Psi) \right. \\ &\quad \left. + 12\ddot{\Psi}(\Phi - \Psi) + 6(\dot{\Phi}\dot{\Psi} + \nabla\Psi \cdot \nabla\Psi) \right] , \\ R^2 &= \varepsilon^2 \left(2\nabla^2\Phi + 6\ddot{\Psi} - 4\nabla^2\Psi \right)^2 , \\ G_0^0 &= -2\varepsilon\nabla^2\Psi , \\ G_i^0 &= -2\varepsilon\nabla_i\dot{\Psi} , \\ G_j^i &= \varepsilon \left[\left(\nabla^i\nabla_j - \frac{1}{3}\delta_j^i\nabla^2 \right) (\Psi - \Phi) \right. \\ &\quad \left. + \frac{2}{3}\delta_j^i (3\ddot{\Psi} + \nabla^2\Phi) \right] , \\ R_\nu^\mu R_\mu^\nu &= G_\nu^\mu G_\mu^\nu \\ &= \varepsilon^2 \left[4(\nabla^2\Psi)^2 - 8\nabla\dot{\Psi} \cdot \nabla\dot{\Psi} \right. \\ &\quad \left. + \varepsilon^2 \{ \nabla_i\nabla_j(\Psi - \Phi) \nabla^i\nabla^j(\Psi - \Phi) \right. \\ &\quad \left. - \frac{2}{3}\nabla^2(\Psi - \Phi) \left[\nabla^2(\Psi - \Phi) - 2(3\ddot{\Psi} + \nabla^2\Phi) \right] \right. \\ &\quad \left. + \frac{1}{3} \left[\nabla^2(\Psi - \Phi) - 2(3\ddot{\Psi} + \nabla^2\Phi) \right]^2 \right] . \end{aligned} \quad (10)$$

Here $\nabla^2 \equiv \delta^{ij}\partial_i\partial_j$ represents the three dimensional Laplace operator and $i, j = 1, 2, 3$. Substituting the above expressions into the action, Eq. (8), and integrating by parts, we obtain the Lagrangian density as follows:

$$\begin{aligned} \mathcal{L} &= -\Lambda^2 g_1 \varepsilon^2 \left(4\nabla\Phi \cdot \nabla\Psi + 6\dot{\Psi}^2 - 2\nabla\Psi \cdot \nabla\Psi \right) \\ &\quad + g_2 \varepsilon^2 \left\{ 4(\nabla^2\Psi)^2 - 8\nabla\dot{\Psi} \cdot \nabla\dot{\Psi} + \frac{4}{3}(3\ddot{\Psi} + \nabla^2\Phi)^2 \right. \\ &\quad \left. + \frac{2}{3}[\nabla^2(\Psi - \Phi)]^2 \right\} \\ &\quad + g_3 \varepsilon^2 \left(2\nabla^2\Phi + 6\ddot{\Psi} - 4\nabla^2\Psi \right)^2 . \end{aligned} \quad (11)$$

A. Ghost-free Constraints

It is convenient to consider the low energy and high energy limits of the Lagrangian density separately, which are given, respectively, by

$$\mathcal{L}_L = -\Lambda^2 g_1 \varepsilon^2 \left(4\nabla\Phi \cdot \nabla\Psi + 6\dot{\Psi}^2 - 2\nabla\Psi \cdot \nabla\Psi \right) , \quad (12)$$

and

$$\begin{aligned} \mathcal{L}_H &= g_2 \varepsilon^2 \left\{ 4(\nabla^2\Psi)^2 - 8\nabla\dot{\Psi} \cdot \nabla\dot{\Psi} + \frac{4}{3}(3\ddot{\Psi} + \nabla^2\Phi)^2 \right. \\ &\quad \left. + \frac{2}{3}[\nabla^2(\Psi - \Phi)]^2 \right\} \\ &\quad + g_3 \varepsilon^2 \left(2\nabla^2\Phi + 6\ddot{\Psi} - 4\nabla^2\Psi \right)^2 . \end{aligned} \quad (13)$$

To have the theory be free of ghosts in all the energy scales, the kinetic terms in both \mathcal{L}_L and \mathcal{L}_H must be positive. This yields,

$$g_1 < 0, \quad g_2 + 3g_3 > 0. \quad (14)$$

B. Constraints from Stability

In order to investigate the stability problem, we shall first derive the equation of motion for the scalar potential Φ and Ψ . To this end, let's substitute Eq. (11) into the Euler-Lagrange equation, then we find

$$\begin{aligned} & -4\Lambda^2 g_1 \nabla^2 \Psi + g_2 \left(\frac{4}{3} \nabla^4 \Psi - 8 \nabla^2 \ddot{\Psi} - 4 \nabla^4 \Phi \right) \\ & + g_3 \left(16 \nabla^4 \Psi - 8 \nabla^4 \Phi - 24 \nabla^2 \ddot{\Psi} \right) = 0, \end{aligned} \quad (15)$$

and

$$\begin{aligned} & g_2 \left(\frac{28}{3} \nabla^4 \Psi - 16 \nabla^2 \ddot{\Psi} + 24 \ddot{\ddot{\Psi}} + 8 \nabla^2 \ddot{\Phi} - \frac{4}{3} \nabla^4 \Phi \right) \\ & + g_3 \left(72 \ddot{\ddot{\Psi}} + 32 \nabla^4 \Psi - 16 \nabla^4 \Phi + 24 \nabla^2 \ddot{\Phi} - 96 \nabla^2 \ddot{\Psi} \right) \\ & - 4g_1 \left(\nabla^2 \Psi - 3 \ddot{\Psi} - \nabla^2 \Phi \right) = 0. \end{aligned} \quad (16)$$

From Eq. (15) we obtain

$$\begin{aligned} \nabla^4 \Phi = & \frac{1}{g_2 + 2g_3} \left[-g_1 \nabla^2 \Psi + g_2 \left(\frac{1}{3} \nabla^4 \Psi - 2 \nabla^2 \ddot{\Psi} \right) \right. \\ & \left. + g_3 \left(4 \nabla^4 \Psi - 6 \nabla^2 \ddot{\Psi} \right) \right]. \end{aligned} \quad (17)$$

Substituting this equation into $\nabla^2 \otimes$ Eq. (16), we obtain the equation of motion for Ψ . Then, considering the k mode of its Fourier transformation:

$$\Psi = \int d^3 k d\omega \Psi_k \left(\omega, \vec{k} \right) e^{i(\omega t - \vec{k} \cdot \vec{x})}, \quad (18)$$

we obtain

$$\begin{aligned} & -\lambda^2 g_1 \left\{ 4k^4 - 12\omega^2 k^2 + \frac{4}{g_2 + 2g_3} \left[-\lambda^2 g_1 k^2 \right. \right. \\ & \left. \left. - g_2 \left(\frac{k^4}{3} - 2\omega^2 k^2 \right) + g_3 (6\omega^2 k^2 - 4k^4) \right] \right\} \\ & + g_2 (8k^4 \omega^2 - 24k^2 \omega^4 - 8k^6) \\ & + g_3 (72k^4 \omega^2 - 72\omega^4 k^2 - 16k^6) = 0. \end{aligned} \quad (19)$$

In the low energy limit, terms proportional to g_2 and g_3 are negligible, and the above equation reduces to

$$g_1 k^2 = 0, \quad (20)$$

which is exactly the result of Poisson equation for Newtonian gravity in Minkowski spacetime:

$$\nabla^2 \Psi = 0. \quad (21)$$

In the high energy limit, terms proportional to g_1 are negligible, and the above equation turns out to be

$$\omega^4 + Bk^2 \omega^2 + Ak^4 = 0, \quad (22)$$

where B and A are defined as

$$\begin{aligned} B &= -\frac{g_2 + 9g_3}{3(g_2 + 3g_3)}, \\ A &= \frac{g_2 + 2g_3}{3(g_2 + 3g_3)}. \end{aligned} \quad (23)$$

Then we obtain from Eq. (22)

$$\omega_{\pm}^2 = \frac{k^2}{2} \left(-B \pm \sqrt{B^2 - 4A} \right). \quad (24)$$

To study the stability problem, it is found convenient to consider the cases, $B = 0$, $B > 0$, and $B < 0$ separately.

1. $B = 0$

When $B = 0$, we divide it further into five sub-cases.

1) $B = 0$, $A > 0$: This implies that

$$g_3 > 0, \quad g_2 = -9g_3, \quad (25)$$

or

$$g_3 < 0, \quad g_2 = -9g_3. \quad (26)$$

For the first solution given by Eq.(25), it can be shown that there is no ghost. But the corresponding theory is unstable, since now we have $\Delta \equiv B^2 - 4A < 0$. For the second solution, there are both ghost and instability in the theory.

2) $B = 0$, $A = 0$: This is a trivial case, in which we have

$$g_2 = 0, \quad g_3 = 0. \quad (27)$$

Thus, the corresponding theory is Einstein's.

3) $B = 0$, $A < 0$: In this case, it can be shown that there is no solution.

2. $B > 0$

Similar to the last case, we further divide it into the following sub-cases.

1) $B > 0$, $A > 0$, $\Delta > 0$: Then, we find the solutions are given by,

$$g_3 > 0, \quad -\frac{3}{11} \left(7 + 2\sqrt{15} \right) g_3 < g_2 < -3g_3, \quad (28)$$

or

$$g_3 < 0, \quad -\frac{3}{11} \left(7 + 2\sqrt{15} \right) g_3 > g_2 > -3g_3. \quad (29)$$

For the first solution, there is no ghost. But the theory is unstable, since ω is pure imaginary. For the second solution, there are both ghost and instability.

2) $B > 0$, $A > 0$, $\Delta = 0$: In this case, we find the solution is given by,

$$g_3 > 0, \quad g_2 = -\frac{3}{11} (7 + 2\sqrt{15}) g_3. \quad (30)$$

Then, there is no ghost. But the theory is unstable since ω is pure imaginary.

3) $B > 0$, $A > 0$, $\Delta < 0$: We find the solution is given by:

$$g_3 > 0, \quad -9g_3 < g_2 < -\frac{3}{11} (7 + 2\sqrt{15}) g_3. \quad (31)$$

In this case, there is no ghost. But the theory is unstable because of $\Delta < 0$.

4) $B > 0$, $A = 0$: It can be shown that in this case there is no solution.

5) $B > 0$, $A < 0$: We find the solutions are given by:

$$g_3 > 0, \quad -3g_3 < g_2 < -2g_3, \quad (32)$$

or

$$g_3 < 0, \quad -3g_3 > g_2 > -2g_3, \quad (33)$$

For the first solution, there is no ghost. Furthermore, the theory is stable. For the second solution, the theory is stable but not ghost-free.

3. $B < 0$

In this case, we can also divide it into five sub-cases.

1) $B < 0$, $A > 0$, $\Delta > 0$: Then, we find the solutions are given by,

$$g_3 > 0, \quad -\frac{3}{11} (7 - 2\sqrt{15}) g_3 > g_2 > -2g_3, \quad (34)$$

or

$$g_3 < 0, \quad -\frac{3}{11} (7 - 2\sqrt{15}) g_3 < g_2 < -2g_3. \quad (35)$$

For the first solution, the theory is both stable and ghost-free. For the second solution, the theory is stable but is not ghost-free.

2) $B < 0$, $A > 0$, $\Delta = 0$: In this case, we find the solutions are given by,

$$g_3 > 0, \quad g_2 = -\frac{3}{11} (7 - 2\sqrt{15}) g_3, \quad (36)$$

or

$$g_3 < 0, \quad g_2 = -\frac{3}{11} (7 + 2\sqrt{15}) g_3. \quad (37)$$

For the first solution, the theory is both stable and ghost-free. For the second solution, the theory is stable but is not ghost-free.

3) $B < 0$, $A > 0$, $\Delta < 0$: Then, we find the solutions are given by:

$$g_3 > 0, \quad g_2 < -9g_3, \quad (38)$$

$$g_3 > 0, \quad g_2 > -\frac{3}{11} (7 - 2\sqrt{15}) g_3, \quad (39)$$

$$g_3 < 0, \quad g_2 > -9g_3, \quad (40)$$

and

$$g_3 < 0, \quad g_2 < -\frac{3}{11} (7 - 2\sqrt{15}) g_3. \quad (41)$$

Among the four solutions, two of them are ghost-free. But the four solutions are all unstable because of $\Delta < 0$.

4) $B < 0$, $A = 0$: In this case, we find the solutions are given by,

$$g_3 > 0, \quad g_2 = -2g_3, \quad (42)$$

or

$$g_3 > 0, \quad g_2 = -2g_3. \quad (43)$$

For the first solution, the theory is both stable and ghost-free. For the second solution, the theory is stable but is not ghost-free.

5) $B < 0$, $A < 0$: Then, we find the solutions are given by,

$$g_3 > 0, \quad -3g_3 < g_2 < -2g_3, \quad (44)$$

and

$$g_3 < 0, \quad -2g_3 < g_2 < -3g_3. \quad (45)$$

For the first solution, the theory is both stable and ghost-free. For the second solution, the theory is stable but is not ghost-free.

Combining all the above discussions, we obtain the following constraints for the theory to be both ghost-free and stable,

$$\begin{aligned} g_1 &< 0, \\ g_3 &> 0, \\ -3g_3 &< g_2 \leq -\frac{3}{11} (7 - 2\sqrt{15}) g_3. \end{aligned} \quad (46)$$

Turning to the numerical results, Eq. (7), which are related our constants as $g_i = g_{i*}$, ($i = 1, 2, 3$), we find that Eq. (7) does satisfy the above constraints.

IV. CONCLUSION AND DISCUSSION

In this paper, we have investigated the problems of ghosts and stability in the framework of the asymptotically safe theory of gravity, and found explicitly the conditions (46), with which the theory is ghost-free and stable. These conditions are satisfied by the newly-obtained

values of the coupling constants [20], given by Eq.(7) at the fixed point. Therefore, it is concluded that the asymptotical safe theory of gravity is both stable and ghost-free. Actually, Ref. [24] argues that the asymptotic gravity contains ghosts only if the gravitational constant decays in the weak regime. On the contrary, if the gravitational constant does not decay in the weak regime there are probably no ghost.

It should be noted that strictly speaking our above conclusion is true only for the effective action with one loop corrections. In the following, we shall argue that the conditions (46) equally hold when high loop corrections are taken into account. To show this clearly, let us first concentrate ourselves on the second loop corrections, which add nine independent six-order derivative terms to the effective action [3], and in Eq.(8) we only presented explicitly two of them, represented by the dimensionless coupling constants g_5 and g_6 , because these are the only terms that are relevant to the ghost and instability problems. All the rest is of third order of ε , as one can see from the expressions given by Eq.(10), in which we find that $R_{ij} \simeq \mathcal{O}(\varepsilon)$ in the Minkowski background. Therefore, they have no contributions to the quadratic action. The $g_{5,6}$ terms, on the other hand, are six-order derivatives, which are dominant only when the energy is sufficiently high, say, higher than E_{6th} , that is, $\omega, k \gg E_{6th}$. In such a high energy scale, the requirements for the theory to be ghost-free and stable will impose conditions only on the coupling constants g_5 and g_6 , as the relevant terms proportional to $g_{1,2,3}$ are all negligible, provided that no fine-tuning exists. As the energy decreases, these terms become less and less important, and finally the terms

proportional to g_2 and g_3 become dominant. Then, the conditions for the theory to be free of ghosts and instability are given precisely by Eq.(46) for g_2 and g_3 . As the energy continuously decreases, the g_1 terms will finally become dominant, and these requirements then lead to the condition $g_1 < 0$, as given by Eq.(46). For the n th loop, only the quadratic terms like $R\Box^{(n-1)}R$ are relevant to the ghost and instability problems, which are the polynomials of the $2(n+1)$ -order of ω and k . Again, these terms become dominant only at even higher energy scale. Following the same arguments as given above, one can see that they have no effects on the low energy constants, as far as the ghost- and instability-free conditions are concerned. As a result, the conditions (46) hold even when high order corrections are taken into account.

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- [1] G. t Hooft and M. J. G. Veltman, *Annales. Poincare. Phys. Theor. A* **20**, 69 (1974)
 - [2] S. Deser and P. van Nieuwenhuizen, *Phys. Rev. D* **10**, 401 (1974), *Phys. Rev. D* **10**, 411 (1974); *Phys. Rev. Lett.* **32**, 245 (1974); S. Deser, H.-S. Tsao, and P. van Nieuwenhuizen, *Phys. Rev. D* **10**, 3337 (1974).
 - [3] M. H. Goroff and A. Sagnotti, *Phys. Lett. B* **160**, 81 (1985).
 - [4] A. E. M. van de Ven, *Nucl. Phys. B* **378**, 309 (1992).
 - [5] S. M. Christensen and M. J. Duff, *Nucl. Phys. B* **170**, 480 (1980).
 - [6] Y. V. Gryzov, A. Y. Kamenshchik, and I. P. Karmazin, *Russ. Phys. J.* **35**, 201 (1992) (*Izv. VUZ, Fiz.* **29**, 121 (1992)).
 - [7] A. O. Barvinsky, A. Y. Kamenshchik, and I. P. Karmazin, *Phys. Rev. D* **48**, 3677 (1993)
 - [8] S. Weinberg, *Critical Phenomena for Field Theorists, in Understanding the Fundamental Constituents of Matter*, ed. A. Zichichi (Plenum Press, New York, 1977).
 - [9] in *General Relativity*, ed. S. W. Hawking and W. Israel (Cambridge University Press, 1979): 790.
 - [10] H. Kawai, Y. Kitazawa and M. Ninomiya, Ultraviolet stable fixed point and scaling relations in $(2+\epsilon)$ -dimensional quantum gravity, *Nucl. Phys. B* **404**, 684 (1993) [arXiv:hep-th/ 9303123].
 - [11] M. Reuter, Nonperturbative Evolution Equation for Quantum Gravity, *Phys. Rev. D* **57**, 971 (1998)[arXiv:hep-th/9605030].
 - [12] W. Souma, Non-trivial ultraviolet fixed point in quantum gravity, *Prog. Theor. Phys.* **102**, 181 (1999)[arXiv:hep-th/9907027].
 - [13] O. Lauscher and M. Reuter, Ultraviolet fixed point and generalized flow equation of quantum gravity, *Phys. Rev. D* **65**, 025013 (2002) [arXiv:hep-th/0108040].
 - [14] O. Lauscher and M. Reuter, Is quantum Einstein gravity nonperturbatively renormalizable?, *Class. Quan. Grav.* **19**, 483 (2002) [arXiv:hep-th/0110021].
 - [15] D. F. Litim, Fixed points of quantum gravity, *Phys. Rev. Lett.* **92**, 201301 (2004) [arXiv:hep-th/ 0312114].
 - [16] M. Niedermaier, The asymptotic safety scenario in quantum gravity: An introduction, *Class. Quan. Grav.* **24**, R171 (2007) [arXiv:gr-qc/0610018].
 - [17] Y-F Cai and D. A. Easson, Black holes in an asymptotically safe gravity theory with higher derivatives *JCAP* **1009**, 002 (2010) [arXiv:gr-qc/1007.1317].
 - [18] A. Codello, R. Percacci and C. Rahmede, Ultraviolet properties of $f(R)$ -gravity, *Int. J. Mod. Phys. A* **23**, 143 (2008) [arXiv:0705.1769 [hep-th]].

- [19] A. Codello, R. Percacci and C. Rahmede, Investigating the Ultraviolet Properties of Gravity with a Wilsonian Renormalization Group Equation, *Ann. Phys.* **324**, 414 (2009) [arXiv:0805.2909 [hep-th]].
- [20] D. Benedetti, P. F. Machado and F. Saueressig, Asymptotic safety in higher-derivative gravity, *Mod. Phys. Lett. A* **24**, 2233 (2009) [arXiv:0901.2984 [hep-th]].
- [21] S. Weinberg, Living with Infinities, arXiv:0903.0568 [hep-th].
- [22] S. Weinberg, Effective Field Theory, Past and Future, arXiv:0908.1964 [hep-th].
- [23] S. Weinberg, Asymptotically Safe Inflation, *Phys. Rev. D* **81**, 083535 (2010) arXiv:0911.3165 [hep-th].
- [24] G. Dvali, S. Folkerts and C. Germani Physics of Trans-Planckian Gravity, arXiv:1006.0984 [hep-th].